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SOLID GEOMETRY.

BY HOWARD F. HART.

If I should make a culture-epoch-theory investigation into my own experience with a certain theorem I would probably put my experience into three stages as follows:

1. "If the unit of measure for angles is the right angle, the area of a spherical triangle is equal to its spherical excess multiplied by the area of a tri-rectangular spherical triangle."

2. "A spherical triangle equals a lune whose angle is half the spherical excess of the triangle."

$$(3) \quad T_{A, B, C} = L_{\frac{A+B+C-180^\circ}{2}}.$$

Neglecting the differences in form between the first two due to the fact that the first is a theorem of mensuration while the second is a theorem of geometry, we see that the second is a simpler statement than the first of the relation between the surface of a triangle and that of the sphere. The third is an accurate, simpler, symbolized form of the second. In it certainly the proposition has been reduced to its lowest terms.

The device, then, consists in using a notation in which the lune of angle A is denoted by L_A ; the spherical triangle of angles A , B and C by $T_{A, B, C}$; the spherical polygon of angles $A_1 A_2 \dots A_n$ by $P_{A_1 A_2 \dots A_n}$; the spherical wedge of angle A by W_A , etc.

In this notation one group of theorems of spherical geometry is stated as follows:

$$(1) \quad L_A \pm L_B = L_{A \pm B},$$

$$(2) \quad 1/n \cdot L_A = L_{A/n},$$

$$(3) \quad (L_A \cong L_B)_{A=B},$$

$$(4) \quad \frac{L_A}{L_B} = \frac{A}{B},$$

$$(5) \quad L_{2\pi} = 4\pi R^2,$$

$$(6) \quad P_{A_1 A_2 \dots A_n} = L_{\frac{(A_1 + A_2 + \dots + A_n) - (n-2)180^\circ}{2}},$$

Another group might be also given as follows:

- (1) $\frac{W_A}{W_B} = \frac{A}{B}, \quad W_A \pm W_B = W_{A \pm B},$
 $\frac{1}{n} \cdot W_A = W_{A/n},$
- (2) $(W_A \cong W_B)_{A=B},$
- (3) $W_{2\pi} = \frac{4}{3}\pi R^3.$

If also $T_{A,B,C}$ and $T'_{A',B',C'}$ denote polar spherical triangles of sides a, b, c and a', b', c' the same idea can be extended to that group of propositions, viz., "If T and T' are polar then $a =$, in measure, $180^\circ - A$," etc.

I first used the notation in simplifying the proof of the proposition quoted at the beginning of this paper and then with the area group and now wherever it is worth while.

Its chief advantage is its power of simplification. It is that sort of advantage which an open-faced watch case has over an hunting case. The real meaning of a proposition is at once evident.

If you will observe the statement of theorems (1) and (2) in the first group you will see that in addition to the advantages of a symbolic expression we have also those due to an operative system (*e. g.*, the lunes add by adding their angles which fact is indicated by an operation in the symbols).

Suppose that we consider the first theorem to follow after (5) in the area group and prove it.

So. Given $T_{A,B,C}$.

To prove $T_{A,B,C} = L_{\frac{A+B+C-180^\circ}{2}}.$

Proof 1. Extend the sides T to form $L_A, L_B, L_C,$

2. $\widehat{BC'} = \widehat{CB'}, \widehat{AB'} = \widehat{BA'}, \widehat{AC} = \widehat{A'C'} \quad (=^\circ - =^\circ);$
3. $AB'C = BC'A' \quad (3 \text{ sides});$
4. $L_{180^\circ} = L_B + (L_A - T) + (L_C - T) \quad \text{whole} = \text{sum parts};$
5. $2T_{A,B,C} = L_A + L_B + L_C - L_{180} \quad (=^\circ \pm =^\circ);$
6. $2T_{A,B,C} = L_{A+B+C-180^\circ} \quad \text{cf. (1) above};$
7. $T_{A,B,C} = L_{\frac{A+B+C-180}{2}} \quad \text{cf. (2) above.}$

The notation is helpful in solving exercises as the result is

found by merely substituting, *e. g.*, "Find the area of a spherical triangle whose angles are 70° , 80° , 90° on a sphere whose diameter is 20 in." (Hopkins.)

$$T = L_{\frac{70^\circ + 80^\circ + 90^\circ - 180^\circ}{2}} = L_{30^\circ} = \frac{1}{12} L_{2\pi} = \frac{4\pi \cdot 100}{12} = \frac{100\pi}{3} \text{ sq. in.}$$

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CONTENTMENT.

Let not the thoughts of what is yet to come
Mar the sweet blessings that are ours to-day;
How can we doubt, when every living thing
Proves that His hand is leading all the way?

Ne'er yet a sorrow but has left the heart
Stronger and nobler, having felt the pain;
Joy comes again to every troubled breast,
Just as the sunshine follows after rain.

Drink in the sunshine, wear a happy smile;
Brighten the way therein our steps have trod;
Do the kind act and say the cheering word;
Lift up your heart in thankful prayer to God.

So shall the burdens of our life below
Seem so much lighter and our hearts more strong,
If by our words, our acts, our prayers, our deeds
We can make life one perfect, grand, sweet song.